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Erratum: Continuum interpretation of the dynamical-triangulation formulation of quantum Einstein gravity

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Abstract: An error in the numerical evaluation of the lattice-continuum conversion factor affects the magnitude of the continuum curvature in several plots. A corrected plot shows somewhat smaller systematic uncertainties. Another plot that would become less informative after correction is replaced by a more revealing one.

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The continuum curvature $R_{\rm c} = R_{\rm osc}/\lambda^2 = \pm 12/(\lambda r_0)^2$ contains the factor λ that converts the lattice geodesic-distance to the continuum distance. The DOB-fit λ-values 0.405 (0.465) quoted below (4.19) are wrong by a factor $\pi^{-1/4}$, due to an error in a Mathematica code. The correct values averaged over the nearly κ_2-independent values in the crumpled (elongated) phase are 0.3044 (0.3483). Consequently, in the plots depending on the DOB-fit curvature, its $R_{\rm c}$ needs to be multiplied by a factor $\sqrt{\pi} \approx 1.8$. This affects figures[1] 6, 15 and 21. To start with the latter, the corrected version below shows less systematic dependence on the fitting method in the elongated phase, since the DOB-fit $R_{\rm c}$ now lies in-between the $R_{\rm c}$s of the A-fit and B-fit.

The vertical scales in figure 6 and the right plot of figure 15 need to be multiplied by respectively $\sqrt{\pi}$ and $1/\sqrt{\pi}$, which is easily visualized. However, because of the increased difference in magnitude between $R_{\rm c}$ and $R_{\rm av} - R_{\rm s}$, a corrected left plot in figure 15 would become unappealing to the eye and blur the difference in shape between $R_{\rm c}$ and $R_{\rm av}$. Below is an alternative plot showing $R_{\rm c}$ together with a renormalized Regge curvature defined as follows. The renormalizations Z_0 and Z_R are chosen such that $R_{\rm ren} = (R_{\rm av} - Z_0)/Z_R$

[1] And the extrapolated Z_R values 0.20 (0.14) mentioned above figure 15 have to be divided by $\sqrt{\pi}$.

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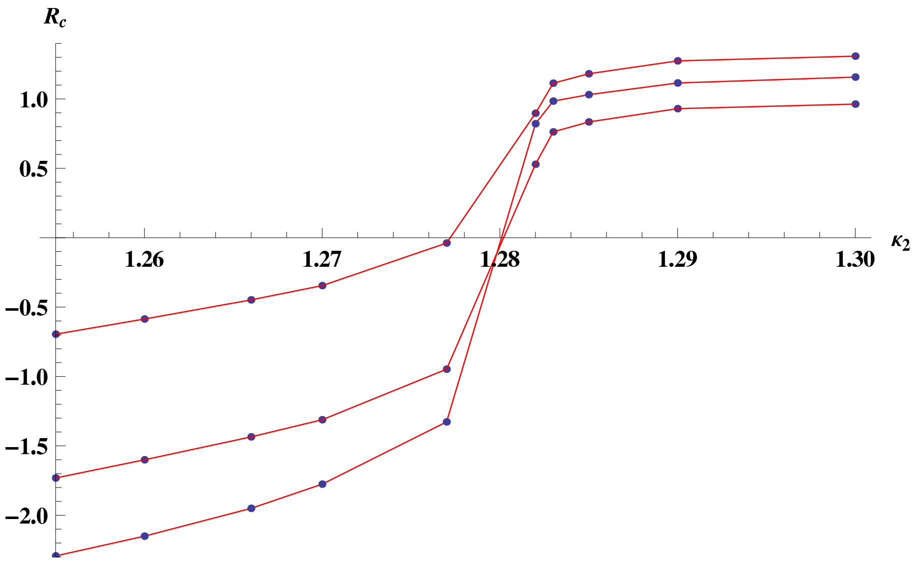

Figure 21. Curvatures $R_{\rm c} = \pm 12/(\lambda r_0)^2$ with linear interpolation. From top to bottom in $\kappa_2 > 1.28$: A-fit, DOB-fit, B-fit.

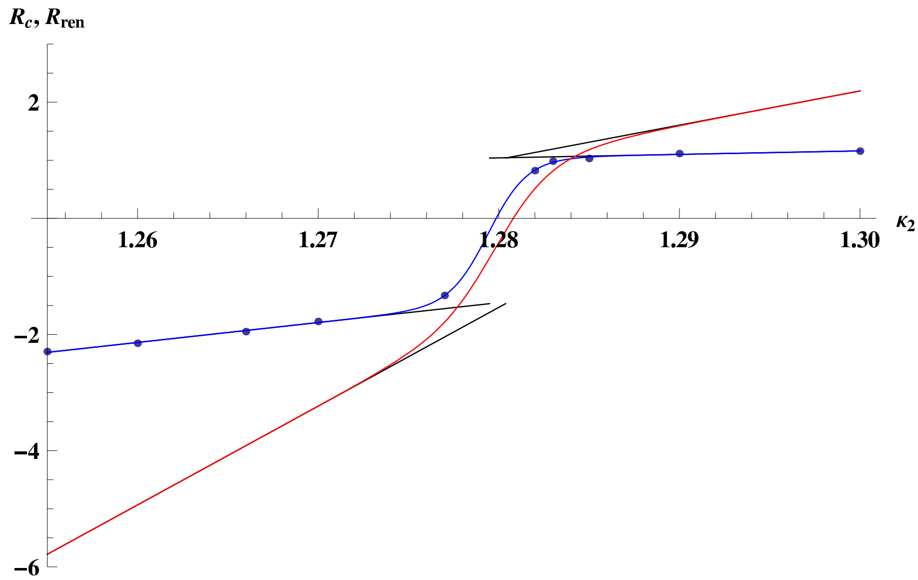

Figure 22. Alternative to a corrected left plot in figure 15. Data points of $R_{\rm c}$ (blue), its fit (8.6) (blue), $R_{\rm ren} = (R_{\rm av} - Z_0)/Z_R$ (red) with $R_{\rm av}$ from the Gaussian-model fit, and extrapolations to their respective $\kappa_2^\star$ (black); $N_4 = 64000$, units $\tilde{\ell} = \ell/\sqrt{10} = 1$.

matches $R_{\rm c}$ when extrapolated to the transition using their fitted functional forms.[2] This means solving the two equations $(R_s \pm R_{\rm d} - Z_0)/Z_R = (R'_{\rm s} \pm R'_{\rm d})/Z'$ for Z_0 and Z_R:

$$Z_0 = R_{\rm s} - R_{\rm d}\frac{R'_{\rm s}}{R'_{\rm d}} \simeq 7.0801\,\tilde{\ell}^{-2}, \quad Z_R = \frac{R_{\rm d}}{R'_{\rm d}}\,Z' \simeq 0.0934. \tag{1}$$

[2]For the record: the fitted parameters of the fitting function (8.6) for $R_{\rm c}$ are $\tilde{\ell}^2 R'_{\rm s} = -0.02904$, $\tilde{\ell}^2 R'_{\rm d} = 0.1691$, $s'_{\rm s} = 148$, $s'_{\rm d} = -104$, $Z' = 0.1349$, $\kappa_2'^{*} = 1.27949$.

Using these renormalizations[3] the resulting $R_{\rm ren}$ is plotted in figure 22 together with $R_{\rm c}$. The difference in slopes away from the transition is clear. In the elongated phase, the value of $R_{\rm c}$ extrapolated to the transition is, in the continuum interpretation, the curvature of an average branched-polymer component, a four-sphere of radius $r_{0\rm c} \simeq 1.075\,\ell$ and volume $\simeq 1510\, v_4$.

Note added: the apparent approach to 1 of the alternative Z_R based on a comparison of $R_{\rm av}$ with the Gauss-Bonnet curvature of the total volume (suggestive of a higher than first-order transition) was a misleading clue, a *red herring*. The approximate volume-independence of the parameters in table (8.4) (except κ_2^*) corroborates a first-order phase transition. Recently, strong evidence for a first-order transition was presented in [2].

References

[1] J. Smit, *Continuum interpretation of the dynamical-triangulation formulation of quantum Einstein gravity*, *JHEP* **08** (2013) 016 [arXiv:1304.6339] [INSPIRE].

[2] T. Rindlisbacher and P. de Forcrand, *Euclidean dynamical triangulation revisited: is the phase transition really 1st order? (extended version)*, *JHEP* **05** (2015) 138 [arXiv:1503.0370] [INSPIRE].

[3] The constant Z_R is close to λ^2 averaged over the two phases. There appears to be no fundamental reason why this should be so. In two dimensions the conversion factor λ for $R_{\rm c}$ is still nontrivial even in flat spacetime (as calculated in appendix A), whereas the volume-averaged Regge curvature needs no renormalization because $\int d^2x\,\sqrt{g}\,R$ is a topological invariant. In two dimensions, the Regge curvature will assign the same value as the Gauss-Bonnet curvature when an SDT configuration of spherical topology is viewed as the limit of a smooth volume-preserving deformation of a two-sphere.